

GENERALIZATION OF AN IDENTITY OF RAMANUJAN

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Dedicated to Prof. A.K. Agarwal on his 70th Birth Anniversary

Abstract: In this paper, an identity of Ramanujan has been generalized. Particular cases of this generalized identity have been discussed.

Keyword and Phrases: Identity, Basic hypergeometric, Basic hypergeometric series of two variables, Basic hypergeometric series of several variables, q-binomial theorem.

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1. Introduction, Notations and Definitions

For complex variables a and q , $|q| < 1$, the q -shifted factorials are given as,

$$(a; q^k)_0 = 1, \quad (a; q^k)_n = (1 - a)(1 - aq^k) \dots (1 - aq^{(n-1)k}),$$

where n and k are positive integers. For brevity, let

$$(a_1, a_2, \dots, a_r; q^k)_n = (a_1; q^k)_n (a_2; q^k)_n \dots (a_r; q^k)_n.$$

Following [4; (1.2.22), p.4], the generalized basic hypergeometric series is defined by

$${}_r\Phi_s \left[\begin{matrix} a_1, a_2, \dots, a_r; q^k; z \end{matrix} \right] = \sum_{n=0}^{\infty} \frac{(a_1, a_2, \dots, a_r; q^k)_n z^n}{(q, b_1, b_2, \dots, b_s; q^k)_n} \{(-1)^n q^{kn(n-1)/2}\}^{1+s-r}. \quad (1.1)$$